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# Local optical characteristics of mixed clouds: Simple parameterizations

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**Abstract**

Simple parameterizations for local scattering and absorption characteristics of mixed clouds are presented. Parameterizations are obtained using Mie theory for droplets and Macke's fractal model for nonspherical ice crystals. The accuracy of the approximation developed is studied using correspondent numerical calculations based on Mie theory and geometrical optics Monte-Carlo calculations for crystals.


*Keywords:* Mixed clouds; Scattering; Absorption

## 1. Introduction

Simple approximate equations are extremely useful in estimations of cloud optical properties and also for the solution of inverse problems. Therefore a number of approximate solutions for local optical cloud characteristics like cloud extinction $\sigma_{\text{ext}}$ and absorption $\sigma_{\text{abs}}$ coefficients, single scattering albedo $\omega_0=1-\sigma_{\text{abs}}/\sigma_{\text{ext}}$, and also the asymmetry parameter $g$ have been derived (van de Hulst, 1981; Ackerman and Stephens, 1987; Mitchell, 2000; Yang et al., 2000; Kokhanovsky, 2004). However, in most of cases either liquid droplets or ice crystals have been considered.

The task of this work is to present parameterizations, which can be used for studies of optical waves propagation in mixed clouds. Formulae derived can be applied to the solution of inverse problems (e.g., particle sizing using spectra of reflected light).

The work is composed of several sections. The next section is devoted to the derivation of an accurate equation for the extinction coefficient of a mixed cloud. Then we consider the probability of photon absorption $\beta=1-\omega_0$ in a local volume of a mixed cloud and also the asymmetry parameter $g$. The values of $\beta$ and $g$ determine the similarity parameter $s=\sqrt{\beta/(1-g\omega_0)}$, which regulates the level of light absorption in a scattering medium (van de Hulst, 1974).

## 2. Extinction coefficient

The extinction coefficient $\sigma_{\text{ext}}$ of a mixed cloud can be presented as the sum of the extinction coefficients for crystals and droplets ($\sigma_{\text{ext}}^{\text{c}}$, $\sigma_{\text{ext}}^{\text{d}}$, respectively). So we have: $\sigma_{\text{ext}}=\sigma_{\text{ext}}^{\text{c}}+\sigma_{\text{ext}}^{\text{d}}$ or

$$\sigma_{\text{ext}}=N_{\text{c}}\langle C_{\text{ext}}^{\text{c}}\rangle+N_{\text{d}}\langle C_{\text{ext}}^{\text{d}}\rangle. \tag{1}$$

Here $\langle C_{\text{ext}}^{\text{c}}\rangle$ is the average extinction cross section of crystals in a cloud and $\langle C_{\text{ext}}^{\text{d}}\rangle$ has the same meaning but

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for droplets, $N_c$ and $N_d$ are the number concentration of crystals and droplets, respectively. The sizes of crystals are much larger than the wavelength of incident visible light (typically, 50–200 μm). This means that the extinction cross section of an individual crystal can be easily found using the fact that

$$C_{ext}^{c} = 2G_c \tag{2}$$

in this case. The value of $G_c$ gives the geometrical cross section of a crystal (van de Hulst, 1981).

The same equation can be used for water droplets (typical diameters 10–30 μm) in visible. However, the accuracy of such an approach decreases in near-infrared spectral region for droplets (van de Hulst, 1981; Kokhanovsky, 2006a). So we need to have a correction factor $f(x)=1+K(x)$ as compared to Eq. (2) for a single droplet:

$$C_{ext}^{d} = 2G_d(1+K(x)), \tag{3}$$

where $x=ka$, $a$ is the radius of a droplet, $k=2\pi/\lambda$, $\lambda$ is the wavelength. The combination of Eqs. (1)–(3) gives:

$$\sigma_{ext} = 2N_c\langle G_c\rangle + 2N_d\langle G_d\rangle\{1+K(x)\}, \tag{4}$$

where subscripts mean crystals (c) or droplets (d). The correction factor ($x$) was studied by many authors (see, e.g., Ackerman and Stephens, 1987). Clearly, it follows: $K(x)\to 0$ as $x\to\infty$ (van de Hulst, 1981). We derive from Eq. (4) using the mean value theorem:

$$\sigma_{ext} = 2N_c\langle G_c\rangle + 2N_d\langle G_d\rangle\{1+K(\bar{x})\}, \tag{5}$$

where $\bar{x}$ is generally an unknown parameter dependent on the droplet size distribution. Eq. (5) can written in yet another form introducing the dimensionless volumetric particle concentration $\phi=N\langle V\rangle$ either for droplets or for ice crystals. Here $\langle V\rangle$ is the average volume of particles. Then it follows from Eq. (5):

$$\sigma_{ext} = \frac{2\phi_c\langle G_c\rangle}{\langle V_c\rangle} + \frac{2\phi_d\langle G_d\rangle\{1+K(\bar{x})\}}{\langle V_d\rangle} \tag{6}$$

or

$$\sigma_{ext} = \frac{3\phi_c}{2a_{ef}^{c}} + \frac{3\phi_d\{1+K(\bar{x})\}}{2a_{ef}^{d}}, \tag{7}$$

where the sign $c$ or $d$ refers to crystals or droplets, respectively, and we introduced the effective radius

$$a_{ef} = \frac{3\langle V\rangle}{4\langle G\rangle}. \tag{8}$$

It is known that the surface area $\Sigma$ is given by the prouct $4\langle G\rangle$ for randomly oriented convex particles (van de Hulst, 1981). So we can write: $a_{ef}=3\langle V\rangle/\Sigma$. Clearly, $a_{ef}$ coincides with the radius of a spherical particle for monodisperse ensembles of spheres. The term $K(\bar{x})$ can be parameterized assuming the following analytical dependence of $K$ on $x_{ef}=ka_{ef}^{d}$:

$$\mathrm{K} = \frac{\mathrm{A}}{x_{ef}^{2/3}} - \mathrm{B}\left(1-\exp\left\{\frac{C}{x_{ef}^{2/3}}\right\}\right). \tag{9}$$

The constants in these equation can be found using the numerical calculations according to the Mie theory (van de Hulst, 1981). In particular, we derive in this way: $A=1.1$, $B=0.0000017$, $C=56.3$. We assumed that the refractive index of droplets $m=1.33$ in our calculations. Also the Deirmendjan's Cloud C1 droplet gamma size distribution with the varied effective droplet radius $a_{ef}^{d}$ was used (Deirmendjian, 1969). The dependence as shown in Eq. (9) is rooted in the complex angular momentum theory as developed by Nussenzweig (2006).

Therefore, we have finally:

$$\sigma_{ext} = \frac{3\phi_c}{2a_{ef}^{c}} + \frac{3\phi_d}{2a_{ef}^{d}}\left[1+\frac{\mathrm{A}}{x_{ef}^{2/3}} - \mathrm{B}\left(1-\exp\left\{\frac{C}{x_{ef}^{2/3}}\right\}\right)\right]. \tag{10}$$

Indices c and d have the same meaning as above. It follows at $\phi_c=0$:

$$\sigma_{ext} = \frac{3\phi_d}{2a_{ef}^{d}}\left[1+\frac{\mathrm{A}}{x_{ef}^{2/3}} - \mathrm{B}\left(1-\exp\left\{\frac{C}{x_{ef}^{2/3}}\right\}\right)\right]. \tag{11}$$

The accuracy of this equation is shown in Fig. 1 at $a_{ef}^{d}=4$ and 16 $\mu m$. It follows that the accuracy is better than 1% at $a_{ef}\geq 4$ $\mu m$ and $\lambda\leq 2.5$ $\mu m$. The extinction is practically spectrally neutral for larger droplets. However, it increases with the wavelength at $a_{ef}=4$ $\mu m$. It is known that the aerosol extinction usually decreases with the wavelength. So we see that the spectral extinction of clouds differs in this respect from that of aerosols in the spectral range studied.

The high accuracy of the corresponding equation for the crystalline clouds is insured by the physics of the problem ($C_{ext}\to 2G$ for large crystals; van de Hulst, 1981).

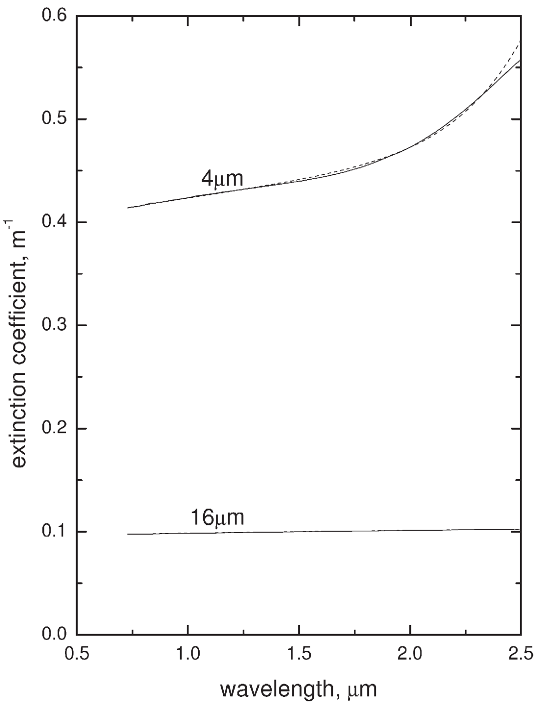


Fig. 1. Dependence of the extinction coefficient on the wavelength for the gamma distribution of water droplets at $\phi_d$=0.000001 calculated using Mie theory (solid lines). The coefficient of variance of the gamma distribution is equal to $7^{-1/2}$ (Deirmendjian, 1969). The effective size of droplets is equal to 4 μm and 16 μm. The approximation is shown by broken lines. The spectral refractive index of water was taken from tables given by Segelstein (1981).

Eq. (10) allows to estimate the spectral extinction of a mixed cloud having different values of effective radii of particles and their concentrations.

## 3. The probability of photon absorption

Generally, it is much harder to derive an accurate analytical equation for the absorption coefficient of a cloud. The same is true for the probability of photon absorption $\beta=\sigma_{abs}/\sigma_{ext}$. This is mostly due to the following reasons. Firstly, the absorption coefficient strongly depends not only on the size of particles as in the case of the extinction coefficient but also on the water bulk absorption coefficient $\alpha=4\pi\chi/\lambda$, where $\chi$ is the imaginary part of the complex refractive index $m=n-i\chi$. The value of $\sigma_{abs}$ is highly sensitive to the load of pollutants in clouds (especially in the visible) and also to the temperature. Secondly, the absorption cross section of a single crystal normalized to $G$ depends on the shape of a crystal. So the unique relationship as presented in Eq. (2) cannot be given in the case of arbitrary absorption.

The probability of photon absorption $\beta_c$ by a single crystal having $a_{ef}$ large as compared to the wavelength $\lambda$ can be parameterized using the following equation (Kokhanovsky and Macke, 1997; Kokhanovsky, 2006b):

$$\beta_c = \beta_\infty(1-\exp\{-\ell/\ell_0\}), \tag{12}$$

where $\beta_\infty$ is the value of $\beta_c$ for a particle which absorbs all rays transmitted through its surface ($\ell/\ell_0\to\infty$), $\ell_0=\lambda/4\pi\chi$ is the ice penetration depth, and the parameter $\ell$ depends both on the shape and the size of a crystal. The dependence of $\ell$ on $\chi$ can be neglected as found by Kokhanovsky and Macke (1997) calculating values of $\beta_c$ for various nonspherical particles using a Monte Carlo ray-tracing technique (see also Kokhanovsky and Nauss, 2005). The dependence of $\ell$ on the real part of the refractive index $n$ must be accounted for. However, $n$ for ice varies little in visible and near IR. So we will use: $n=1.3$.

Eq. (12) can be generalized for the case of ice crystals having different shapes and sizes. Then we have instead of Eq. (12):

$$\langle\beta_c\rangle = \beta_\infty(1-\langle\exp\{-\ell/\ell_0\}\rangle), \tag{13}$$

where we assumed that particles are randomly oriented and have a convex shape. This allows the substitution of $\beta_\infty$ by the value of this parameter for spheres (van de Hulst, 1981). The dependence of $\beta_\infty$ on $n$ for spherical particles is given by Kokhanovsky (2004). It equals to 0.47 at the real part of refractive index $n=1.3$ close to that of ice in visible and near IR. Using, the mean value theorem, we derive:

$$\langle\beta_c\rangle = \beta_\infty(1-\exp\{-\bar{\ell}/\ell_0\}), \tag{14}$$

where $\bar{\ell}$ is the so-called particle absorption length (Kokhanovsky, 2006b). Numerical calculations performed by us show that the dependence of $\bar{\ell}$ on $\chi$ can be neglected. Therefore, Eq. (14) allows for the experimental determination of $\bar{\ell}$ from measurements of $<\beta_c>$ as discussed by Kokhanovsky and Nauss (2005). The relationship between $\bar{\ell}$ and $a_{ef}^c$ for crystals of various shapes must be a subject of future work. In particular, Kokhanovsky (2006b) found using geometrical optics Monte-Carlo calculations that

$$\bar{\ell} = \xi a_{ef}^c \tag{15}$$

$\xi=1.8$ for ice spheres and $\xi=3.6$ for fractal (see, e.g., Macke et al., 1996) particles. The experimental determination of $\xi$ and also the study of its variability

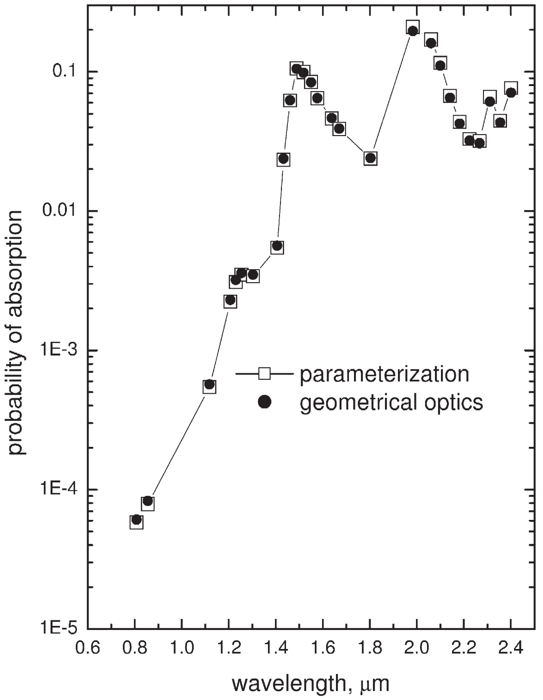


Fig. 2. Spectral dependence of the probability of photon absorption $\beta=1-\omega_0$ for fractal ice particles according to the parameterization and geometrical optics results. The effective radius of the fractal particle is equal to 46 μm. The spectral refractive index of ice was taken from tables given by Warren (1984).

for real world ice clouds is of a considerable importance. For parameterization purposes we may write:

$$\langle\beta_c\rangle = \beta_\infty(1-\exp\{-\xi a_{ef}^c/\ell_0\}), \tag{16}$$

where ξ depends on the shape of particles as discussed above. In particular, results for fractals can be used for estimations of ξ. The accuracy of Eq. (16) for fractal ice particles is shown in Fig. 2 as compared to time consuming Monte-Carlo ray-tracing calculations. The error is below 7.5% at $\lambda \le 2.4$ μm. Maxima shown in Fig. 2 correspond to maxima of light absorption coefficient of ice.

It is easy to show that it follows for a mixed cloud:

$$\beta = \frac{\sigma_{ext}^c\langle\beta_c\rangle + \sigma_{ext}^d\langle\beta_d\rangle}{\sigma_{ext}^c + \sigma_{ext}^d}, \tag{17}$$

where $\langle\beta_c\rangle$ and $\langle\beta_d\rangle$ must be calculated using Eq. (16) (but with different values of ξ as indicated above). This allows us to find the absorption extinction coefficient as well:

$$\sigma_{abs} = \langle\beta_c\rangle\sigma_{ext}^c + \langle\beta_d\rangle\sigma_{ext}^d. \tag{18}$$

Results for $\langle\beta_c\rangle$ and $\langle\beta_d\rangle$ are accurate only for radii much larger than $\lambda$. This is not a problem for crystals due to their large sizes. However, droplets in clouds (especially in the near infrared) may not obey this condition. Then $\langle\beta_d\rangle$ in Eqs. (17), (18) must be substituted by $\langle\beta_d\rangle=\sigma_{abs}^d/\sigma_{ext}^d$, where

$$\sigma_{abs}^d = \alpha\phi_d F(a_{ef}^d) \tag{19}$$

with

$$F(a_{ef}^d) = 1.23(1-1.3\alpha a_{ef}^d)[1 + 0.34\{1-\exp(-8\lambda/a_{ef}^d)\}]. \tag{20}$$

Constants in Eq. (20) were obtained by the parameterization of numerical results based on Mie theory for the droplet size distributions described above. The general dependence of $F$ on the effective radius as shown in Eq. (20) was proposed by Kokhanovsky (2004).

It follows as $\alpha a_{ef}^d \to 0$ for large particles: $F\to 1.23$ and $\sigma_{abs}^d \to 1.23\alpha\phi_d$. The accuracy of Eq. (19) is shown in Figs. 3 and 4. It is better than 10% at $\lambda \le 2.4$ μm. We also show errors for $\sigma_{ext}^d$ and $\sigma_{sca}^d=\sigma_{ext}^d-\sigma_{abs}^d$ on the

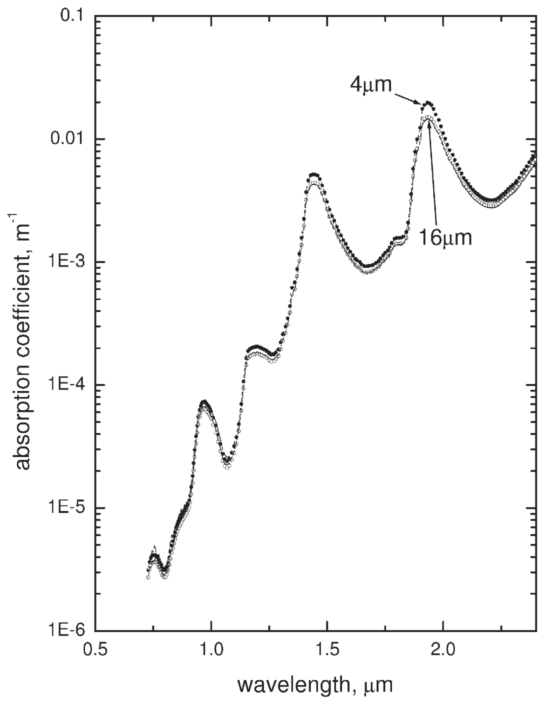


Fig. 3. Dependence of the absorption coefficient on the wavelength for the gamma distribution of water droplets at $\phi_d=0.000001$ calculated using Mie theory (solid lines). The coefficient of variance of the gamma distribution is equal to $7^{-1/2}$ (Deirmendjian, 1969). The effective size of droplets is equal to 4 μm and 16 μm. The approximation is shown by symbols. The spectral refractive index of water was taken from tables given by Segelstein (1981).

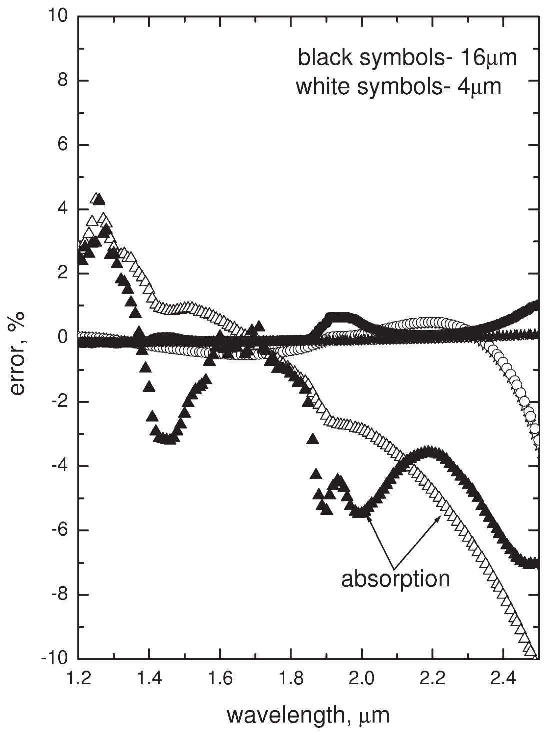


Fig. 4. Relative errors of approximations for the absorption (triangles), scattering (circles) and extinction (stars) coefficients of clouds with liquid water droplets.

same figure. Errors for extinction $\sigma_{\text{ext}}^{\text{d}}$ and scattering $\sigma_{\text{sca}}^{\text{d}}$ coefficients almost coincide at $a_{\text{ef}}=4$ μm. The error for the scattering coefficient is slightly larger than that for the extinction coefficient at $a_{\text{ef}}=16$ μm (see Fig. 4).

This allows us to derive the following parameterization for the probability of photon absorption by an elementary volume of a mixed cloud:

$$\beta = M\beta_{\infty}(1-\exp(-\xi a_{\text{ef}}^{\text{c}}/\ell_0)) + PF(a_{\text{ef}}^{\text{d}}), \tag{21}$$

where $M=\sigma_{\text{ext}}^{\text{c}}/\sigma_{\text{ext}}$, $P=\alpha\phi_{\text{d}}/\sigma_{\text{ext}}$.

## 4. The asymmetry parameter

It is known that the radiative properties of clouds are highly sensitive (van de Hulst, 1974; Kokhanovsky, 2004, 2006a) to the asymmetry parameter defined as

$$g = \frac{1}{2}\int_0^{\pi} p(\theta)\sin\theta\cos\theta\mathrm{d}\theta, \tag{22}$$

where $p(\theta)$ is the phase function. The phase function describes the angular distribution of scattered light by a given volume of a light scattering medium. It follows for a mixed cloud:

$$g = \frac{g^{\text{c}}\sigma_{\text{sca}}^{\text{c}} + g^{\text{d}}\sigma_{\text{sca}}^{\text{d}}}{\sigma_{\text{sca}}^{\text{c}} + \sigma_{\text{sca}}^{\text{d}}}. \tag{23}$$

Scattering coefficients can be easily found from equations for $\beta$ and $\sigma_{\text{ext}}$. Namely, it follows: $\sigma_{\text{sca}}=(1-\beta)\sigma_{\text{ext}}$.

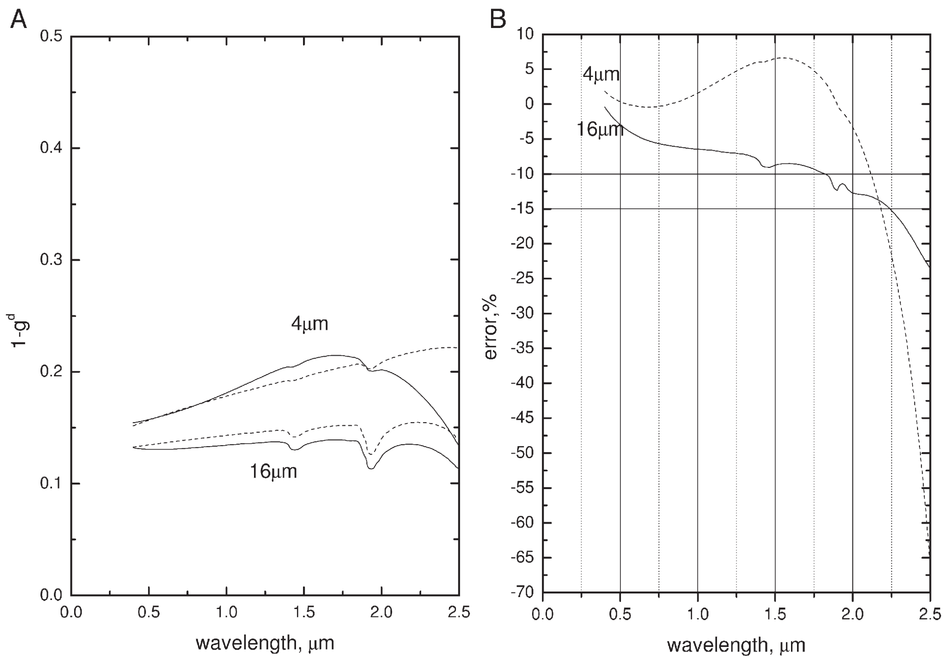


Fig. 5. (A) The same as in Fig. 3 but for $1-g^{\text{d}}$. (B) Relative errors calculated using data shown in (A).

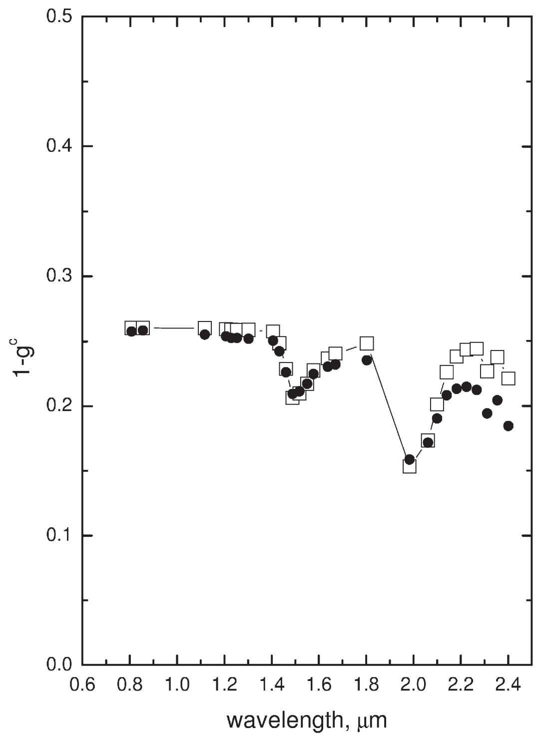


Fig. 6. The same as in Fig. 2 except for $1-g^c$.

The asymmetry parameter for droplets $g^d$ can be parameterized as follows using the same procedure as outlined above for $\beta$ (Kokhanovsky, 2004):

$$g^d = 0.88 - 0.15(ka^d_{ef})^{-2/3} + 0.15\alpha a^d_{ef}. \tag{24}$$

The accuracy of this formula is shown in Fig. 5A,B (with respect to the symmetry parameter $\varepsilon^d = 1 - g^d$). It follows that the error of Eq. (24) for finding $\varepsilon^d = 1 - g^d$ is below 10% at $\lambda \leq 1.8$ μm and $a^d_{ef} = 4$, 16 μm. The error sharply increases at $\lambda = 2.2$ μm because geometrical optics assumptions used in the derivation of Eq. (24) are not valid at $a^d_{ef} = 4$ μm for longer wavelengths. The error reaches 22% at $a_{ef} = 16$ μm and $\lambda = 2.5$ μm. The correspondent errors for the calculation of $g^d$ itself are much smaller. The error is below 5% for $\varepsilon^d$ at $\lambda \leq 1.6$ μm.

The general equation for the asymmetry parameter of the crystalline media $g^c$ is difficult to obtain due to poor information on the representative habits of crystals in mixed and ice clouds. Also the asymmetry parameter is highly dependent on the shape. This means that for every shape yet another parameterization must be used as outlined by Yang et al. (2000). The question arises which shape is the most representative and, therefore, which parameterization must be chosen. The answer on this question can be obtained only from measurements in natural ice clouds. In particular, Gerber et al. (2000) and Garett et al. (2001) have found that $g^c$ in visible does not vary very much from cloud to cloud and usually is close to 0.75. Taking into account the high variability of shapes in ice clouds and also the small variation of $g^c$, we may conclude that various shapes presented in ice clouds produce a scattering diagram characteristic for a random particle, e.g., a fractal. Calculations by Macke et al. (1996) show that the value of $g$ for randomly oriented fractal particles is 0.74, which is close to measurements. So we propose to use the fractal particle model in conjunction with Monte-Carlo geometrical optics calculations of the asymmetry parameter for different $a^c_{ef}$ for the parameterization of the asymmetry parameter of clouds. This gives:

$$g^c = 0.98 - 0.24\exp(-\xi a^c_{ef}/\ell_0), \tag{25}$$

where $\xi = 3.6$. The error of this approximation (except for $\varepsilon^c = 1 - g^c$) is shown in Figs. 6 and 7. It follows that the error for the calculation of $\varepsilon^c$ using Eq. (25) is below 20% at $\lambda \leq 2.4$ μm. It is better than 6% at $\lambda \leq 2$ μm. The error for the probability of photon

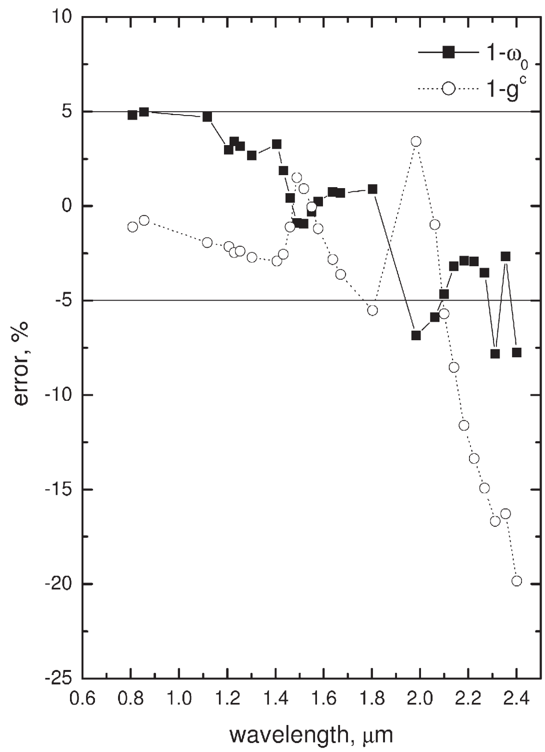


Fig. 7. Relative errors correspondent to data shown in Figs. 2, 6.

absorption $\beta_c$ (as obtained from Fig. 2) is also given in Fig. 7. The error is generally below 5%.

Summing up, it follows for the asymmetry parameter of a mixed cloud:

$$g = \bar{M}(0.98-0.24\exp(-\xi a_{ef}^{c}/\ell_0) + \bar{P}(0.88-0.15(ka_{ef}^{d})^{-2/3} + 0.15\alpha a_{ef}^{d}), \quad (26)$$

where $\bar{M}=\sigma_{sca}^{c}/(\sigma_{sca}^{c}+\sigma_{sca}^{d})$, $\bar{P}=\sigma_{sca}^{d}/(\sigma_{sca}^{c}+\sigma_{sca}^{d})$.

## 5. Conclusion

We propose here simple and yet accurate approximate analytical equations for modeling of light scattering and absorption characteristics of mixed clouds. They can be used in global circulation models and also for rapid estimations of cloud effects on the propagation of optical radiation. If a higher accuracy is needed, then one must use time consuming numerical calculations (e.g., with the Mie theory or ray-tracing approaches).

Results for droplets are based on the parameterization of numerical calculations using the Mie theory. Derived equations allow to find a corresponding light absorption or scattering characteristic assuming effective radii and concentrations. For crystalline clouds, this is not enough. Also the information on the average shape of crystals is needed. Such information is not available *a priori*. Therefore, we assume that a high diversity of crystal habits make the use of the randomized particle (e.g., fractal) model justified. The parameterizations for crystals presented in this paper are based on the fractal particle model as described by Macke et al. (1996). The approximations presented here have been implemented (as a speeding option) in the radiative transfer code SCIATRAN (Rozanov et al., 2005) freely available at www.iup.physik.uni-bremen.de/sciatran.

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